# Equivalent Transformations for Mixed-Lumped and Multiconductor Coupled Circuits

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***Abstract*—Distributed circuits consisting of a cascade connection of $m$-port stub circuits and multiconductor coupled transmission lines are equivalent to ones consisting of cascade connections of multiconductor coupled transmission lines whose characteristic impedances are different from original ones, $m$-port stub circuits, and an $m$-port ideal transformer bank. Because of the reciprocity of the circuit, values of transformer ratio must be identified. In the special case of a one conductor transmission line, these equivalent transformations are equivalent to Kuroda's identities. These extended equivalent transformations may be applied to mixed-lumped and multiconductor coupled circuits. By using these equivalent transformations, equivalent circuits and exact network functions of multiconductor nonuniform coupled transmission lines can be obtained.**

## I. Introduction

We SHOWED that Kuroda's identities are extended to mixed-lumped and nonuniform distributed circuits [5], [7]. By applying these equivalent transformations to nonuniform transmission lines whose exact network functions are known, network functions of many nonuniform transmission lines can be obtained exactly. We also showed that telegrapher's equations of binomial form nonuniform coupled transmission lines can be solved exactly, and equivalent circuits of these nonuniform coupled transmission lines are expressed as mixed-lumped and distributed coupled circuits [6].

In this paper, it is shown that $m$-port networks consisting of stub circuits and multiconductor transmission lines are equivalent to $m$-port networks consisting of cascade connections of multiconductor transmission lines, stub circuits, and an ideal transformer bank, under certain conditions. Then, by applying these equivalent transformations $n$-times to $m$-port distributed circuits and proceeding to the limit $n \to \infty$, these equivalent transformations may be applied to mixed-lumped and multiconductor coupled circuits. Therefore, equivalent circuits and exact network functions of multiconductor nonuniform coupled transmission lines can be obtained. Finally, numerical examples of these equivalent transformations are presented.

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## II. Equivalent Transformations

We define the $m$-port circuit consisting of short-circuited stubs (SCS) in Fig. 1(a), and the $m$-port circuit consisting of open-circuited stubs (OCS) in Fig. 1(b). Here $L_{ij}^{(I)}$ $(i, j = 1,2,\cdots,m;\ I = 0,1,\cdots,n)$ are characteristic impedances of short stubs of length $l/n$, and $C_{ij}^{(I)}$ $(i, j = 1,2,\cdots,m;\ I = 0,1,\cdots,n)$ are characteristic admittances of open stubs of length $l/n$.

Distributed circuits consisting of a cascade connection of the $m$-port SCS and multiconductor coupled circuits, shown in Fig. 2(a), are equivalent to ones consisting of cascade connections of multiconductor coupled transmission lines (CTL) whose characteristic impedances are different from the original ones, an $m$-port SCS, and an $m$-port ideal transformer bank, shown in Fig. 2(b), under the following condition:

$$\left[\frac{1}{L^{(0)}}\right][W^{(1)}] = A[E] \qquad (A = \text{constant}) \tag{1}$$

where $[1/L^{(0)}]$ is the $m \times m$ matrix defined as

$$\left[\frac{1}{L^{(0)}}\right] = \begin{bmatrix} \left(\sum_{i=1}^{m} \frac{1}{L_{1i}^{(0)}}\right) & -\frac{1}{L_{12}^{(0)}} & \cdots & -\frac{1}{L_{1m}^{(0)}} \\ -\frac{1}{L_{21}^{(0)}} & \left(\sum_{i=1}^{m} \frac{1}{L_{2i}^{(0)}}\right) & \cdots & -\frac{1}{L_{2m}^{(0)}} \\ \cdots & & & \\ -\frac{1}{L_{m1}^{(0)}} & -\frac{1}{L_{m2}^{(0)}} & \cdots & \left(\sum_{i=1}^{m} \frac{1}{L_{mi}^{(0)}}\right) \end{bmatrix} \tag{2}$$

and $[W^{(1)}]$ is the $m \times m$ characteristic impedance matrix of the $m$-wire CTL

$$[W^{(1)}] = \begin{bmatrix} W_{11}^{(1)} & \cdots & W_{1m}^{(1)} \\ \vdots & & \vdots \\ W_{m1}^{(1)} & \cdots & W_{mm}^{(1)} \end{bmatrix} \tag{3}$$

and $[E]$ is the $m \times m$ identity matrix. In Fig. 2

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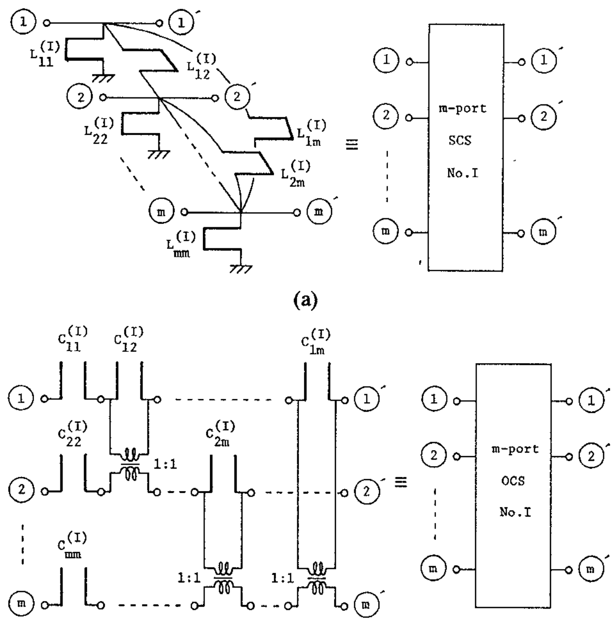


Fig. 1. $m$-port stub circuits. (a) $m$-port short-circuited stub circuits. (b) $m$-port open-circuited stub circuits.

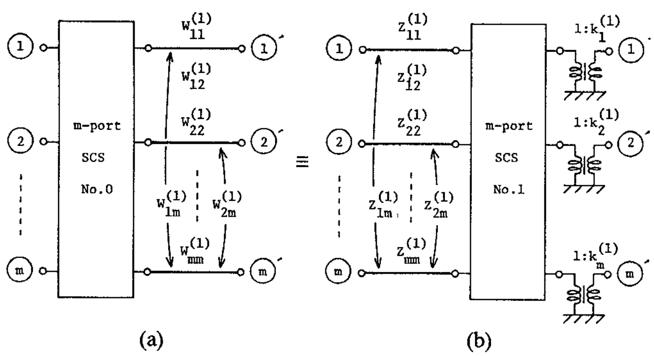


Fig. 2. Equivalent transformation for multiconductor coupled circuits. (a) Original circuit. (b) Equivalent circuit.

| | |
|---|---|
| $W_{ij}^{(1)}$ and $Z_{ij}^{(1)}$ | characteristic impedances of coupled transmission lines of length $l/n$ $(i, j=1,2,\cdots,m)$; |
| $k_i^{(1)}$ | transformer ratio of ideal transformer $(i=1,2,\cdots,m)$. |

From (1), (2), and (3), we can obtain

$$\frac{1}{L_{11}^{(0)}}=A\left(Y_{11}^{(1)}+Y_{12}^{(1)}+\cdots+Y_{1m}^{(1)}\right) \qquad (4)$$

where

$$[Y^{(1)}]=[W^{(1)}]^{-1}=\begin{bmatrix} Y_{11}^{(1)} & \cdots & Y_{1m}^{(1)} \\ \vdots & & \vdots \\ Y_{m1}^{(1)} & \cdots & Y_{mm}^{(1)} \end{bmatrix}. \qquad (5)$$

The condition (1) means that if $[W^{(1)}]$ is given, the constant $A$ is decided with only $[W^{(1)}]$ and $L_{11}^{(0)}$, so $L_{ij}^{(0)}$ are

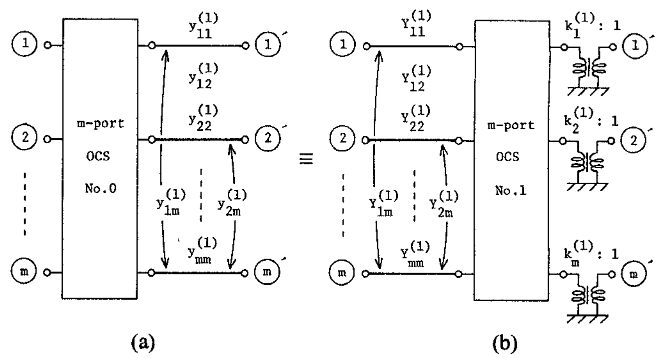


Fig. 3. Equivalent transformation for multiconductor coupled circuits, the dual of the circuit shown in Fig. 2. (a) Original circuit. (b) Equivalent circuit.

related with $A$ and $W_{ij}$ as follows:

$$\frac{1}{L_{ii}^{(0)}}=A\cdot\sum_{j=1}^{m}Y_{ij}^{(1)} \qquad (i=2,3,\cdots,m) \qquad (6)$$

$$\frac{1}{L_{ij}^{(0)}}=A\cdot Y_{ij}^{(1)} \qquad (i\neq j;\, i, j=1,2,\cdots,m). \qquad (7)$$

Element values of the equivalent circuit, shown in Fig. 2(b), are obtained as follows (see Appendix):

$$k^{(1)}\equiv k_1^{(1)}=k_2^{(1)}=\cdots=k_m^{(1)}$$
$$=1+\sum_{i=1}^{m}\frac{W_{1i}^{(1)}}{L_{ii}^{(0)}}=1+\sum_{i=1}^{m}\frac{W_{2i}^{(1)}}{L_{ii}^{(0)}}=\cdots=1+\sum_{i=1}^{m}\frac{W_{mi}^{(1)}}{L_{ii}^{(0)}} \qquad (8)$$

$$Z_{ij}^{(1)}=\frac{W_{ij}^{(1)}}{k^{(1)}} \qquad (i, j=1,2,\cdots,m) \qquad (9)$$

$$L_{ij}^{(1)}=\frac{L_{ij}^{(0)}}{k^{(1)}} \qquad (i, j=1,2,\cdots,m). \qquad (10)$$

Similarly, for the circuit consisting of an $m$-port OCS and a multiconductor CTL shown in Fig. 3, we can obtain the equivalent transformation under the following condition:

$$\left[\frac{1}{C^{(0)}}\right][y^{(1)}]=B[E] \qquad (B=\text{constant}) \qquad (11)$$

where $[1/C^{(0)}]$ is the $m\times m$ matrix given by

$$\left[\frac{1}{C^{(0)}}\right]=\begin{bmatrix} \left(\sum_{i=1}^{m}\frac{1}{C_{1i}^{(0)}}\right) & \cdots & -\frac{1}{C_{1m}^{(0)}} \\ \vdots & & \vdots \\ -\frac{1}{C_{m1}^{(0)}} & \cdots & \left(\sum_{i=1}^{m}\frac{1}{C_{mi}^{(0)}}\right) \end{bmatrix} \qquad (12)$$

and $[y^{(1)}]$ is the $m\times m$ characteristic admittance matrix of

$m$-wire CTL

$$[y^{(1)}]=\begin{bmatrix} y_{11}^{(1)} & \cdots & y_{1m}^{(1)} \\ \vdots & & \vdots \\ y_{m1}^{(1)} & \cdots & y_{mm}^{(1)} \end{bmatrix} \tag{13}$$

and

$y_{ij}^{(1)}$ and $Y_{ij}^{(1)}$ characteristic admittances of coupled transmission lines of length $l/n$ $(i, j=1,2,\cdots,m)$.

In this case, the condition (11) means that if all elements of $[y^{(1)}]$ are given, the characteristic admittances $C_{ij}^{(0)}$ of $(m(m+1)/2-1)$ stubs are uniquely determined from a characteristic admittance of one stub and $[y^{(1)}]$. Element values of the equivalent circuit, shown in Fig. 3(b), are obtained as follows:

$$k^{(1)}\equiv k_1^{(1)}=k_2^{(1)}=\cdots=k_m^{(1)}$$
$$=1+\sum_{i=1}^{m}\frac{y_{1i}^{(1)}}{C_{ii}^{(0)}}=1+\sum_{i=1}^{m}\frac{y_{2i}^{(1)}}{C_{ii}^{(0)}}=\cdots=1+\sum_{i=1}^{m}\frac{y_{mi}^{(1)}}{C_{ii}^{(0)}} \tag{14}$$

$$Y_{ij}^{(1)}=\frac{y_{ij}^{(1)}}{k^{(1)}} \qquad (i,j=1,2,\cdots,m) \tag{15}$$

$$C_{ij}^{(1)}=\frac{C_{ij}^{(0)}}{k^{(1)}} \qquad (i,j=1,2,\cdots,m). \tag{16}$$

The equivalent transformations in Figs. 2 and 3 have a dual relation. In the special case of $m=1$, these equivalent transformations are Kuroda's identities.

Next, we apply $n$-times the equivalent transformation in Fig. 2 to the circuit shown in Fig. 4(a). The transformed circuit is the one consisting of cascade connections of multiconductor CTL, $m$-port SCS, and $m$-port ideal transformer bank as shown in Fig. 4(b). We can obtain the following equations after applying the equivalent transformation $I$-times, i.e.,

$$\begin{bmatrix} [E] & [0] \\ \left[\frac{1}{L^{(I-1)}}\right]\cdot\frac{1}{p} & [E]\end{bmatrix}\begin{bmatrix}[K^{(I-1)}]^{-1} & [0] \\ [0] & [K^{(I-1)}]\end{bmatrix}\begin{bmatrix}[E] & [W^{(I)}]p \\ [W^{(I)}]^{-1}\cdot p & [E]\end{bmatrix}$$
$$=\begin{bmatrix}[E] & [Z^{(I)}]p \\ [Z^{(I)}]^{-1}\cdot p & [E]\end{bmatrix}\begin{bmatrix}[E] & [0] \\ \left[\frac{1}{L^{(I)}}\right]\cdot\frac{1}{p} & [E]\end{bmatrix}\begin{bmatrix}[K^{(I)}]^{-1} & [0] \\ [0] & [K^{(I)}]\end{bmatrix} \qquad (I=1,2,\cdots,n) \tag{17}$$

where

$$[K^{(I)}]=\begin{bmatrix} k_1^{(I)} & 0 & \cdots & 0 \\ 0 & k_2^{(I)} & \cdots & 0 \\ \vdots & \ddots & & \\ 0 & 0 & \cdots & k_m^{(I)}\end{bmatrix}. \tag{18}$$

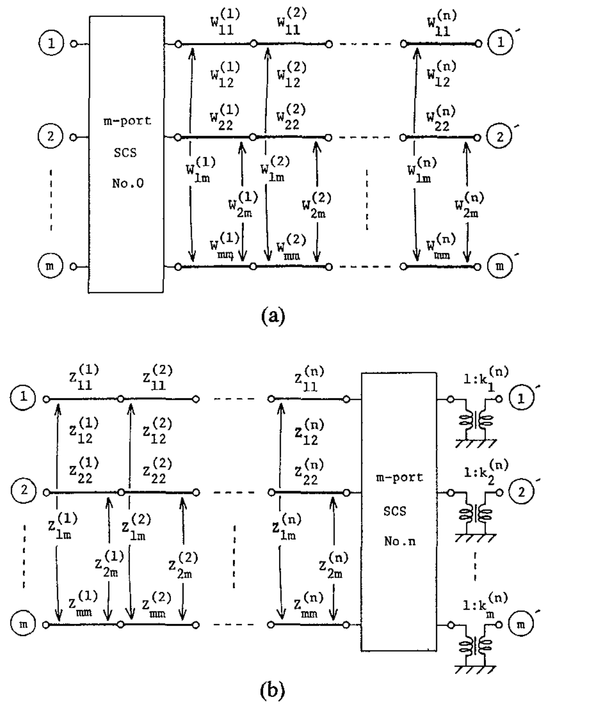


Fig. 4. Equivalent transformation for multiconductor coupled circuits. (a) Original circuit. (b) Equivalent circuit.

$[0]$ is the $m\times m$ zero matrix, and $[Z^{(I)}]$ is the $m\times m$ characteristic impedance matrix of $I$th unit section (US) of the $m$-wire CTL. From (17), we obtain the following equations:

$$[Z^{(I)}][K^{(I)}]=[K^{(I-1)}]^{-1}[W^{(I)}] \qquad (I=1,2,\cdots,n) \tag{19}$$

$$\left[\frac{1}{L^{(I)}}\right][K^{(I)}]^{-1}=\left[\frac{1}{L^{(I-1)}}\right][K^{(I-1)}]^{-1} \qquad (I=1,2,\cdots,n) \tag{20}$$

and

$$[K^{(I)}]=[K^{(I-1)}]+\left[\frac{1}{L^{(I-1)}}\right][K^{(I-1)}]^{-1}[W^{(I)}] \qquad (I=1,2,\cdots,n) \tag{21}$$

where $[K^{(0)}]$ is the $m\times m$ identity matrix. From (19) and

(20), we obtain

$$k_1^{(I)} = k_2^{(I)} = \cdots = k_m^{(I)} \equiv k^{(I)} \qquad (I=1,2,\cdots,n). \tag{22}$$

From (19), (20), and (21), the element values obtained after applying equivalent transformation $I$-times are given as follows:

$$\begin{aligned} k^{(I)} &= 1 + \frac{1}{L_{11}^{(0)}}\cdot\sum_{r=1}^{I} W_{11}^{(r)} + \frac{1}{L_{22}^{(0)}}\cdot\sum_{r=1}^{I} W_{12}^{(r)} + \cdots + \frac{1}{L_{mm}^{(0)}} \\ &\quad \cdot \sum_{r=1}^{I} W_{1m}^{(r)} = 1 + \sum_{s=1}^{m}\left(\frac{1}{L_{ss}^{(0)}}\cdot\sum_{r=1}^{I} W_{1s}^{(r)}\right) \\ &= \cdots = 1 + \sum_{s=1}^{m}\left(\frac{1}{L_{ss}^{(0)}}\cdot\sum_{r=1}^{I} W_{ms}^{(r)}\right) \qquad (I=1,2,\cdots,n) \end{aligned} \tag{23}$$

$$Z_{rs}^{(I)} = \frac{W_{rs}^{(I)}}{k^{(I-1)}\cdot k^{(I)}} \qquad (r,s=1,2,\cdots,m;\ I=1,2,\cdots,n) \tag{24}$$

$$L_{rs}^{(I)} = \frac{L_{rs}^{(0)}}{k^{(I)}} \qquad (r,s=1,2,\cdots,m;\ I=1,2,\cdots,n). \tag{25}$$

## III. Equivalent Transformations for Mixed-Lumped and Multiconductor Coupled Circuits

We define the characteristic impedances $W_{rs}^{(I)}$ of the $I$th US of the multiconductor CTL, shown in Fig. 4(a), as follows [7]:

$$\begin{aligned} W_{rs}^{(I)} = W_{rs}^{(0)}\cdot&\left[1 + a_1\left(\frac{I-1}{n}\right) + a_2\left(\frac{I-1}{n}\right)^2 \right. \\ &\left. + \cdots + a_k\left(\frac{I-1}{n}\right)^k + \cdots\right] \\ &(r,s=1,2,\cdots,m;\ I=1,2,\cdots,n) \end{aligned} \tag{26}$$

where $W_{rs}^{(0)}$ $(r,s=1,2,\cdots,m)$ are characteristic impedances of the first US's of the multiconductor CTL and $a_k$ $(k=1,2,\cdots)$ are constants. Also, the coordinates $x$ of the $I$th US is given by [5]

$$x = \frac{I}{n} l. \tag{27}$$

By substituting (27) in (26) and proceeding to the limit $n \to \infty$, various characteristic impedance distributions $W_{rs}(x)$ of multiconductor nonuniform coupled transmission lines (NCTL) may be expressed by the following equations:

$$W_{rs}(x) = \lim_{n\to\infty} W_{rs}^{(I)} \equiv W_{rs}^{(0)}\cdot f(x) \qquad (r,s=1,2,\cdots,m) \tag{28}$$

where $f(x)$ is a nonuniform taper function

$$f(x) = 1 + a_1\left(\frac{x}{l}\right) + a_2\left(\frac{x}{l}\right)^2 + \cdots + a_k\left(\frac{x}{l}\right)^k + \cdots. \tag{29}$$

By substituting (26) in (23), we obtain

$$\begin{aligned} k^{(I)} = 1 + \sum_{s=1}^{m} \Bigg\| \frac{W_{rs}^{(0)}}{L_{ss}^{(0)}}\cdot&\left[I + \frac{a_1}{n}\left\{\frac{(I-1)^2}{2} + \frac{(I-1)}{2}\right\}\right. \\ &+ \frac{a_2}{n^2}\left\{\frac{(I-1)^3}{3} + \frac{(I-1)^2}{2} + \cdots\right\} \\ &\left.+ \cdots + \frac{a_k}{n^k}\left\{\frac{(I-1)^{k+1}}{k+1} + \frac{(I-1)^k}{2} + \cdots\right\} + \cdots\right]\Bigg\| \\ &(r=1,2,\cdots,m). \end{aligned} \tag{30}$$

Let

$$L_{rs}^{(0)} = nL_{rs} \qquad (r,s=1,2,\cdots,m). \tag{31}$$

By substituting (27) and (31) in (30) and proceeding to the limit $n \to \infty$, we obtain [7]

$$k(x) = \lim_{n\to\infty} k^{(I)} = 1 + \sum_{s=1}^{m}\left[\frac{W_{rs}^{(0)}}{L_{ss}}\int_0^{x/l} f(\lambda)\, d\left(\frac{\lambda}{l}\right)\right] \qquad (r=1,2,\cdots,m) \tag{32}$$

and

$$k = k(x)|_{x=l}. \tag{33}$$

From (24), (28), and (32), characteristic impedance distributions $Z_{rs}(x)$ of transformed multiconductor NCTL become as follows:

$$Z_{rs}(x) = \lim_{n\to\infty} Z_{rs}^{(I)} = \frac{W_{rs}(x)}{k(x)^2} \qquad (r,s=1,2,\cdots,m). \tag{34}$$

Under this condition, the impedances of SCS shown in Fig. 4(a) and (b), respectively, yield

$$\left.\begin{aligned} \lim_{n\to\infty}\left(jnL_{rs}\cdot\tan\frac{\beta l}{n}\right) &= jL_{rs}\cdot\beta l \\ \lim_{n\to\infty}\left(j\frac{nL_{rs}}{k}\cdot\tan\frac{\beta l}{n}\right) &= j\frac{L_{rs}}{k}\cdot\beta l \end{aligned}\right\} \qquad (r,s=1,2,\cdots,m). \tag{35}$$

Namely, in the limit case $n \to \infty$, short-circuited stubs of length $l/n$ become lumped inductances. Therefore, the equivalent transformation in Fig. 2 may be extended to mixed-lumped and multiconductor coupled circuits shown in Fig. 5(a), and the transformed circuit becomes one shown in Fig. 5(b). If characteristic impedance distributions $W_{rs}(x)$ of multiconductor NCTL are given, because of the reciprocity of the original and transformed circuits, the $k$ value of a transformer ratio may be expressed with

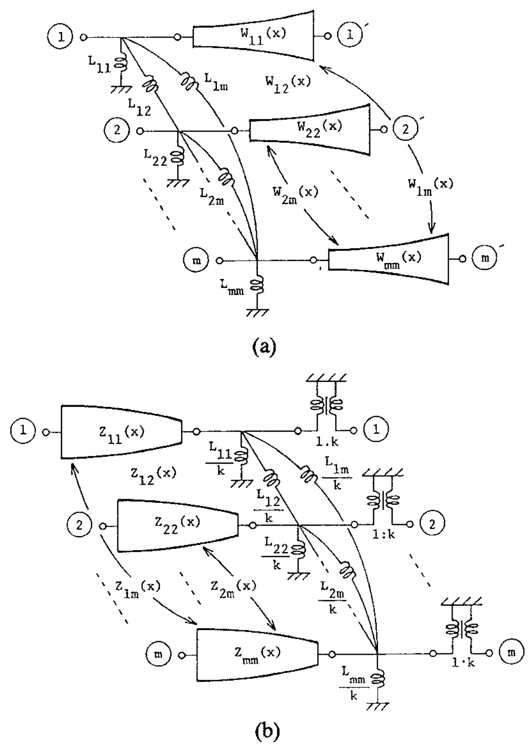


Fig. 5. Equivalent transformation for mixed-lumped and multiconductor nonuniform coupled circuits.

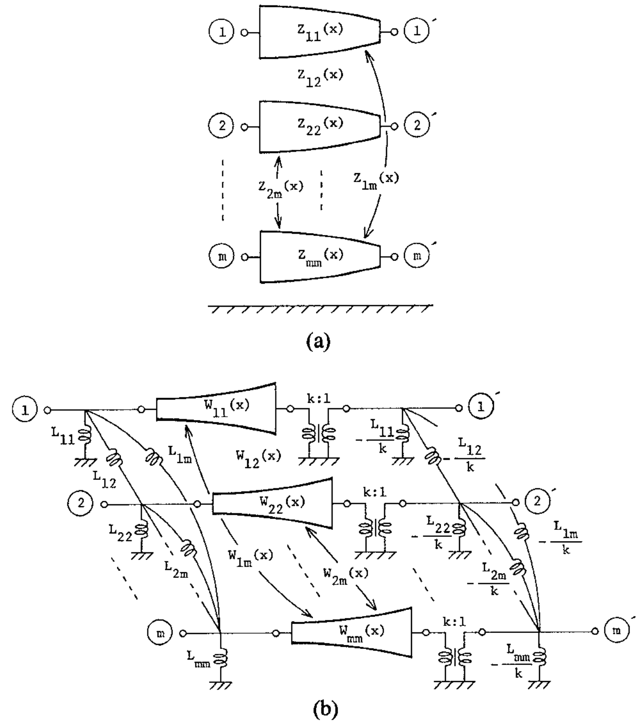


Fig. 6. Multiconductor nonuniform coupled transmission lines and its equivalent circuit.

these characteristic impedance distributions $W_{rs}(x)$ $(r,s=1,2,\cdots,m)$ and one lumped inductance, for example

$$k=1+\frac{\int_0^1 f(x)\,d\left(\frac{x}{l}\right)}{L_{11}\left(Y_{11}^{(0)}+Y_{12}^{(0)}+\cdots+Y_{1m}^{(0)}\right)} \tag{36}$$

where

$Y_{ii}^{(0)}$ self characteristic admittance of $i$th transmission line at $x=0$; and

$Y_{ij}^{(0)}$ mutual characteristic admittance between $i$th and $j$th transmission lines at $x=0$ $\quad (i,j=1,2,\cdots,m)$.

Values of lumped inductances except $L_{11}$ are determined from the following equation uniquely:

$$\left[\frac{1}{L}\right]=\frac{k-1}{\int_0^1 f(x)\,d\left(\frac{x}{l}\right)}\cdot\left[W^{(0)}\right]^{-1} \tag{37}$$

where $[1/L]$ is the same expression as (2) with the $L_{ij}^{(0)}$ values changed to $L_{ij}$, and $[W^{(0)}]$ is the same expression as (3) with the $W_{ij}^{(1)}$ values changed to $W_{ij}^{(0)}$. From Fig. 5(a) and (b), the equivalent circuit of multiconductor NCTL with $Z_{rs}(x)$ of (34), shown in Fig. 6(a), can be expressed as mixed-lumped and multiconductor nonuniform coupled circuits shown in Fig. 6(b). Accordingly, if an exact network function of the original multiconductor NCTL is known, a network function of the transformed multiconductor NCTL can be obtained exactly.

In the same manner, the equivalent transformation in Fig. 3 may be extended to the circuit shown in Fig. 7(a), the dual of the circuit shown in Fig. 5(a). The transformed circuit becomes one shown in Fig. 7(b). The element values

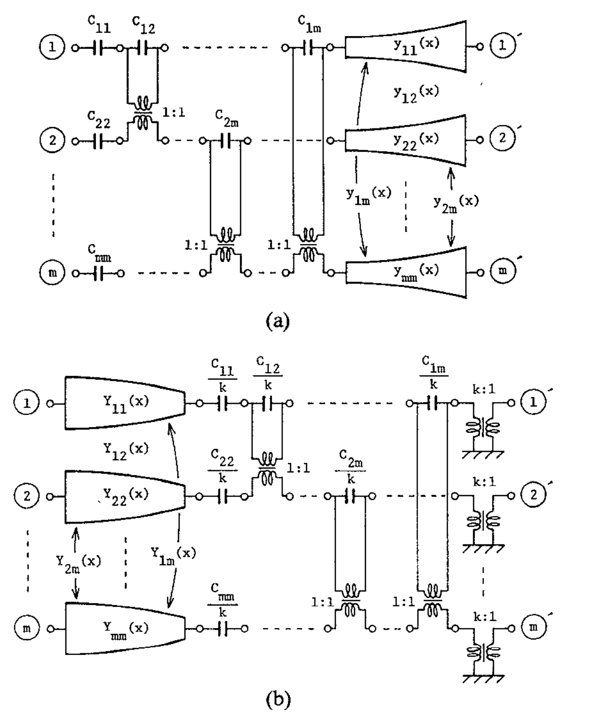


Fig. 7. Equivalent transformation for mixed-lumped and multiconductor nonuniform coupled circuits, the dual of the circuit shown in Fig. 5.

of the transformed circuit are given by

$$k(x)=1+\sum_{s=1}^{m}\left[\frac{y_{rs}^{(0)}}{C_{ss}}\int_0^{x/l} g(\lambda)\,d\left(\frac{\lambda}{l}\right)\right] \qquad (r=1,2,\cdots,m) \tag{38}$$

$$k=k(x)|_{x=l} \tag{39}$$

and

$$Y_{rs}(x)=\frac{y_{rs}(x)}{k(x)^2} \qquad (r,s=1,2,\cdots,m) \tag{40}$$

where $y_{rs}(x)$ and $Y_{rs}(x)$ are characteristic admittance distributions of the original and transformed multiconductor NCTL, respectively, and $y_{rs}(x)$ are expressed by

$$y_{rs}(x)=y_{rs}^{(0)}\cdot g(x) \qquad (r,s=1,2,\cdots,m) \tag{41}$$

where $y_{rs}^{(0)}$ are characteristic admittances between $r$th and $s$th transmission lines at $x=0$, and $g(x)$ is a nonuniform taper function which is the same expression as (29). In this case, the $k$ value of the transformer ratio is given with characteristic admittance distribution $y_{rs}(x)$ $(r, s=1,2,\cdots,m)$ of the $m$-wire NCTL and one lumped capacitor, for example

$$k=1+\frac{\int_0^1 g(x)\,d\left(\frac{x}{l}\right)}{C_{11}\left(Z_{11}^{(0)}+Z_{12}^{(0)}+\cdots+Z_{1m}^{(0)}\right)} \tag{42}$$

where

$$\overset{m\times m}{[Z^{(0)}]}=\overset{m\times m}{[y^{(0)}]}^{-1}. \tag{43}$$

From Fig. 7(a) and (b), the equivalent circuit of a multiconductor NCTL with $Y_{rs}(x)$ of (40) can be expressed as mixed-lumped and multiconductor nonuniform coupled circuits, the dual of the circuit shown in Fig. 6(b).

## IV. Examples

*Example 1. A circuit consisting of lumped inductance coupled circuits and 3-wire coupled transmission lines*

Let

$$[W]=\begin{bmatrix} W_{11} & W_{12} & W_{13}\\ W_{21} & W_{22} & W_{23}\\ W_{31} & W_{32} & W_{33}\end{bmatrix}=\begin{bmatrix} 1.0 & 0.2 & 0.1\\ 0.2 & 2.0 & 0.3\\ 0.1 & 0.3 & 3.0\end{bmatrix} \tag{44}$$

and

$$L_{11}=0.556. \tag{45}$$

In order to satisfy the condition of (1), the remaining lumped inductances $L_{ij}$ are determined from the inverse matrix of (44) as follows:

$$\begin{matrix} L_{12}=5.07 & L_{13}=20.7 & L_{23}=10.3\\ L_{22}=1.35 & L_{33}=1.88. & \end{matrix} \tag{46}$$

By substituting (44), (45), and (46) in (32), $k(x)$ is obtained by

$$k(x)=1+\left(\frac{W_{11}}{L_{11}}+\frac{W_{12}}{L_{22}}+\frac{W_{13}}{L_{33}}\right)\frac{x}{l}=1+2\frac{x}{l} \tag{47}$$

and the $k$ value of transformer ratio is given

$$k=k(x)|_{x=l}=3.0. \tag{48}$$

Therefore, characteristic impedance distributions $Z_{ij}(x)$ of the transformed 3-wire NCTL are obtained as follows:

$$Z_{ij}(x)=\frac{W_{ij}}{k(x)^2}=\frac{W_{ij}}{\left(1+2\frac{x}{l}\right)^2} \qquad (i,j=1,2,\cdots,m). \tag{49}$$

*Example 2. A circuit consisting of lumped inductance coupled circuits and 3-wire coupled exponential transmission lines*

Let

$$[W(x)]=\begin{bmatrix} W_{11}(x) & W_{12}(x) & W_{13}(x)\\ W_{21}(x) & W_{22}(x) & W_{23}(x)\\ W_{31}(x) & W_{32}(x) & W_{33}(x)\end{bmatrix}=\exp(\delta x)\cdot\begin{bmatrix} 1.0 & 0.2 & 0.1\\ 0.2 & 1.0 & 0.2\\ 0.1 & 0.2 & 1.0\end{bmatrix} \tag{50}$$

$$\delta l=2.0 \tag{51}$$

and

$$L_{11}=4.07. \tag{52}$$

From (37), the remaining lumped inductances $L_{ij}$ are determined as follows:

$$\begin{matrix} L_{12}=16.3 & L_{13}=48.9 & L_{23}=16.3\\ L_{22}=4.65 & L_{33}=4.07. & \end{matrix} \tag{53}$$

By substituting (50)–(53) in (32), $k(x)$ is obtained by

$$k(x)=1+\frac{\exp(\delta x)-1}{\delta l}\cdot\left(\frac{W_{11}^{(0)}}{L_{11}}+\frac{W_{12}^{(0)}}{L_{22}}+\frac{W_{13}^{(0)}}{L_{33}}\right)=1+\frac{\exp(\delta x)-1}{6.39} \tag{54}$$

and the $k$ value of transformer ratio is given

$$k=k(x)|_{x=l}=2.0. \tag{55}$$

Accordingly, characteristic impedance distributions $Z_{ij}(x)$ of the transformed 3-wire NCTL are given as follows:

$$Z_{ij}(x)=\frac{W_{ij}(x)}{k(x)^2}=W_{ij}^{(0)}\frac{\exp(\delta x)}{(0.844+0.156\exp(\delta x))^2} \qquad (i,j=1,2,3). \tag{56}$$

It is quite difficult to solve a telegrapher's equation of a 3-wire NCTL with $Z_{ij}(x)$ of (56), but, by using the equivalent transformations described here, an exact network function can be easily obtained from the equivalent circuit.

## V. Conclusion

We have applied equivalent transformations to distributed coupled circuits consisting of a cascade connection of $m$-port stub circuits and multiconductor CTL. In this case, because of the reciprocity, all values of transformer ratios must be identified. Namely, if all elements of a multiconductor CTL are given, there is only one free choice for the element values of the $m$-port stub circuits. In the special case of $m=1$, these equivalent transformations reduced to Kuroda's identities. Then, by considering the limit case, these equivalent transformations are extended to the case of mixed-lumped and multiconductor nonuniform coupled circuits. Equivalent circuits of transformed multiconductor NCTL may be represented as mixed-lumped and distributed coupled circuits consisting of cascade connections of $m$-port lumped reactance circuits, the original multiconductor NCTL, and an $m$-port ideal transformer bank. Therefore, if an exact network function of the original multiconductor NCTL is known, a network function of the transformed multiconductor NCTL can be obtained exactly.

## Appendix

Because the circuits of Fig. 2(a) and (b) are equal, the following equation is applicable:

$$\begin{bmatrix} [E] & [0] \\ \left[\dfrac{1}{L^{(0)}}\right]\cdot\dfrac{1}{p} & [E] \end{bmatrix}\begin{bmatrix} [E] & [W^{(1)}]\cdot p \\ [W^{(1)}]^{-1}\cdot p & [E] \end{bmatrix} = \begin{bmatrix} [E] & [Z^{(1)}]\cdot p \\ [Z^{(1)}]^{-1}\cdot p & [E] \end{bmatrix}\begin{bmatrix} [E] & [0] \\ \left[\dfrac{1}{L^{(1)}}\right]\cdot\dfrac{1}{p} & [E] \end{bmatrix} \cdot \begin{bmatrix} [K^{(1)}]^{-1} & [0] \\ [0] & [K^{(1)}] \end{bmatrix} \tag{A-1}$$

where

$p$ $j\tan\beta l/n$: Richards variable,
$\beta$ phase constant,
$l/n$ a commensurate length of the network,

and $[K^{(1)}]$ is the $m\times m$ diagonal matrix. Therefore, the following equations are satisfied:

$$[Z^{(1)}][K^{(1)}]=[W^{(1)}] \tag{A-2}$$

$$\left[\frac{1}{L^{(1)}}\right][K^{(1)}]^{-1}=\left[\frac{1}{L^{(0)}}\right] \tag{A-3}$$

$$[K^{(1)}]=[E]+\left[\frac{1}{L^{(0)}}\right][W^{(1)}]. \tag{A-4}$$

In (A-2), as matrices $[Z^{(1)}]$ and $[W^{(1)}]$ are symmetric, respectively, so it is necessary that all values of transformer ratio are equal:

$$k_1^{(1)}=k_2^{(1)}=\cdots=k_m^{(1)}\equiv k^{(1)}. \tag{A-5}$$

Therefore, from (A-5) and (A-4), the $k^{(1)}$ value of transformer ratio is given by

$$k^{(1)}=1+\sum_{i=1}^{m}\frac{W_{1i}^{(1)}}{L_{ii}^{(0)}}=1+\sum_{i=1}^{m}\frac{W_{2i}^{(1)}}{L_{ii}^{(0)}}=\cdots=1+\sum_{i=1}^{m}\frac{W_{mi}^{(1)}}{L_{ii}^{(0)}}. \tag{A-6}$$

From (A-5) and (A-2), the characteristic impedances $Z_{ij}^{(1)}$ of the $m$-wire CTL are given by

$$Z_{ij}^{(1)}=\frac{W_{ij}^{(1)}}{k^{(1)}} \qquad (i,j=1,2,\cdots,m). \tag{A-7}$$

From (A-5) and (A-3), the characteristic impedances $L_{ij}^{(1)}$ of the $m$-port SCS are given as follows:

$$L_{ij}^{(1)}=\frac{L_{ij}^{(0)}}{k^{(1)}} \qquad (i,j=1,2,\cdots,m). \tag{A-8}$$

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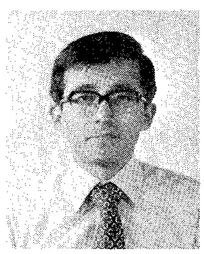

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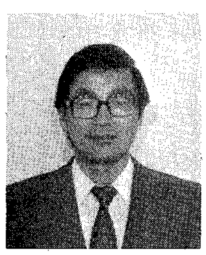

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# Extracted-Pole Filter Manifold Multiplexing

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***Abstract*—A transformation method is introduced for enabling filters of the extracted-pole variety to be match-multiplexed onto a manifold using standard waveguide multiplexer computer programs. Thus the advantages that accrue from the extracted-pole realization for filters may now be extended to multiplexers, which will be particularly useful in narrow-band high-power low-loss multiplexing applications. The measured performance of a 12-GHz contiguous-channel quadraplexer comprising $TE_{011}$ cavity extracted-pole elliptic filters is presented, demonstrating the very low insertion losses attainable with this form of realization. Since the majority of applications envisaged for this type of multiplexer is in high-power output circuitry, a discussion on thermal aspects is included.**

## I. Introduction

In a previous paper [1], the synthesis procedure for extracted-pole filters[1] was introduced and the measured results of a laboratory model presented. This model demonstrated the advantages of building bandpass filters in this way, which stem chiefly from the single-sign coupling elements throughout the filter that the extracted-pole procedure ensures. For when the coupling elements are uniform in sign, advanced self-equalized pseudo-elliptic characteristics may be realized with a single-layer arrangement of $TE_{01n}$-mode resonance cylindrical cavities. This in turn yields the advantages of a high unloaded $Q$ for optimally low in-band insertion losses, relatively large dimensions for immunity from multipactor effect in space, for high-power handling capability, and insensitivity to manufacturing tolerances. Also, the device has a flat bottom for easy transfer of dissipated heat to a flat cooling plate, if necessary. One of the most important applications foreseen for this type of filter is in high-power low-loss contiguous or near contiguous channel multiplexing. When elliptic function characteristics can be used in such multiplexers, performance may be enhanced even further since the design bandwidths may be made greater and sometimes the degree of filter necessary may be reduced, both of which tend to reduce loss and in-band signal distortion. Particularly, application was envisaged for direct-broadcasting TV satellites using recently developed high-power TWTA's and Klystrons, whose output powers may be as high as 600 W. The aim of this paper is to introduce a

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[1]US Patents pending.

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